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Magnetoexcitons in finite-size semiconductor quantum wells

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Abstract. The spectrum of specific elementary excitations in a many-electron system, i.e. magnetoexcitons, is investigated theoretically. It is shown that in finite-size quantum wells there exist additional branches of magnetoexcitons in comparison with ideal two-dimensional systems. The new branches differ from the known branches in their dispersion laws. The general features of the spectrum for all types of magnetoexciton are obtained.

1. Introduction

The various properties of a two-dimensional electron gas in a strong transverse magnetic field have been intensively investigated in recent years. The most important feature of such a system is that the one-particle energy levels are discrete. The interaction of the electrons at low temperatures leads to the creation of some microscopic states such as an incompressible electronic fluid [1, 2] and Wigner crystal.

For better understanding of the many-particle phenomenon it is important to consider the different aspects of electron–electron interactions. One interesting possibility is the investigation of a specific kind of elementary excitation in the system of interacting electrons, namely magnetoexcitons [3, 4].

The magnetoexciton is the neutral excitation which consists of an electron in an empty level connected with a hole in a filled level. This quasi-particle is the exact eigenstate of the system of two-dimensional electrons for any integer filling factor and may be used as a good approximation otherwise. The magnetoexciton can move transverse to a magnetic field H with a two-dimensional momentum p. It was shown in [3, 4] that the energy gap in the spectrum of magnetoexcitons is determined by the one-electron energy (which is equal to the cyclotron energy $\hbar\omega_c$), but the dispersion law is defined by the electron–electron interaction.

The theory [3, 4] predicts a non-monotonic dependence of the energy E of the magnetoexciton on the momentum p with an additional local minimum at a finite value of p. The existence of such a singularity in the spectrum was confirmed experimentally by Pinczuk *et al* [5].

The theoretical results [3, 4] were obtained for the ideal two-dimensional system, but in real semiconductor heterostructures or quantum wells the electrons are not two-dimensional particles. There also exists a motion along the direction of magnetic

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field $\boldsymbol{H}$ which is determined by the potential of the quantum well. In this situation any one-electron energy level can be described by two quantum numbers: the quantum number for the Landau level, i.e. $n = 0, 1, 2, \ldots$, and the quantum number for the level of size quantization, i.e. $m = 1, 2, 3, \ldots$. Every magnetoexciton corresponds to the excitation of an electron from a filled energy level to an empty energy level. In the ideal two-dimensional system these electron transitions correspond to a change in the quantum number n of the Landau level. In quasi-two-dimensional systems there are some additional branches of magnetoexcitons, connected with a possible change in the quantum number m.

In the present paper the spectrum of the different branches of magnetoexcitons is investigated. These excitations may be classified according to the two quantum numbers in the corresponding electron transition. It is shown that there are three qualitatively different types of magnetoexciton in a quantum well with a finite thickness, which have various dispersion laws. The general features of the spectrum of different magnetoexciton branches are established.

2. General relations

Let us consider the system of quasi-two-dimensional electrons in a strong magnetic field directed perpendicular to the quantum well plane (along the z direction). It is suggested that in the ground state some energy levels are completely occupied by electrons. This means that we have an integer filling factor. The filled levels may correspond to the first few values of both the Landau levels n and the size quantization levels m. The strong-magnetic-field limit where the cyclotron energy $\hbar\omega_c$ is large compared with the Coulomb energy $E_0 = e^2/\epsilon r_0$ will be considered, or in other words

$$r_0 \ll a. \tag{1}$$

Here ϵ is the background dielectric constant, $r_0 = (eH/\hbar c)^{1/2}$ is the magnetic length, $a = \hbar^2\epsilon/\mu e^2$ is the Bohr radius and μ is the electron mass.

The important parameter which characterizes the electron motion in a quantum well is the effective width L of the well. In the present paper, only relatively narrow quantum wells will be considered for which the condition

$$L \ll a \tag{2}$$

is realized.

The relation between L and r_0 may be arbitrary. It should be noted that this relation determines the sequence of energy levels. In a very narrow well where $L < r_0$ the cyclotron energy is smaller than the energy of size quantization. In this situation the filled levels may correspond to the different Landau levels and to the ground level of size quantization. This case is close to the ideal two-dimensional electron system. In the opposite case of thicker wells where $L > r_0$ the filled levels may correspond to the different size quantization levels and to the first Landau level.

When conditions (1) and (2) are satisfied, the energy of electron–electron interaction is smaller than the difference between the energies of one-electron levels. Therefore the quantum numbers of the one-particle problem can be used to describe the system of interacting electrons.

To study the spectrum of magnetoexcitons it is convenient to introduce the creation and annihilation operators a^+_{N,k_y} and a_{N,k_y} for filled levels and similar

operators b^+_{M,k_y} and b_{M,k_y} for empty levels. These operators are classified by the projection of the wavevector k and quantum numbers $N = (n, m)$ or $M = (n'm')$ which include the old quantum numbers n and m. In principle the quantum numbers N or M may also include spin but, in the present paper, only the electron excitations without any change in spin state are considered.

The creation and annihilation operators satisfy the common fermionic commutation rules

$$\begin{aligned} \hat{a}^+_{N,k}\hat{a}_{N',k'} + \hat{a}_{N',k'}\hat{a}^+_{N,k} &= \delta_{NN'}\delta_{kk'} \\ \hat{b}^+_{M,k}\hat{b}_{M',k'} + \hat{b}_{M',k'}\hat{b}^+_{M,k} &= \delta_{MM'}\delta_{kk'} \end{aligned} \tag{3}$$

and the $\hat{a}$ and $\hat{b}$ operators anticommute with each other.

The normalized one-electron wavefunction $\chi(\rho, z)$ may be written in the following way:

$$\chi_N(\rho, z) = \exp(ikx)\,\Phi_n(y + kr_0^2/\hbar)\psi_m(z) \tag{4}$$

where Φ_n are the oscillator functions and $\psi_m(z)$ are the wavefunctions of the electron motion in a quantum well. The Landau gauge $A_y = H_x e/c$ is used.

The interaction Hamiltonian H_{int} may be written, according to [4, 6], in the following way:

$$\begin{aligned} U_{\text{int}} = \sum_{k_1k_2}\int \frac{\mathrm{d}^2q}{2\pi}\exp[\mathrm{i}q_x(k_1 - k_2)]\Bigg(&\tfrac{1}{2}\sum_{NN'N_1N_1'} V_{NN';N_1N_1'}(q)\hat{a}^+_{N,k_1^+}\hat{a}^+_{N_1,k_2^-} \\ &\times \hat{a}_{N_1',k_2^+}\hat{a}_{N',k_1^-} + \sum_{NN',MM'}[V_{NN';MM'}(q) - \check{V}_{NM;N'M'}(q)] \\ &\times \hat{a}^+_{N,k_1^+}\hat{b}^+_{M,k_2^-}\hat{b}_{M'k_2^+}\hat{a}_{N',k_1^-}\Bigg). \end{aligned} \tag{5}$$

Here

$$V_{NN';N_1N_1'}(q) = E_0[(1/q)L_{nn'}(q)L^*_{n_1n_1'}(q)J_{mm';m_1m_1'}(q)] \tag{6}$$

where

$$L_{nn'}(q) = \int \mathrm{d}^2\rho\exp(\mathrm{i}q_x\rho)\Phi_n\left(\rho + \frac{q_x r_0^2}{2}\right)\Phi_{n'}\left(\rho - \frac{q_x r_0^2}{2}\right) \tag{7}$$

$$J_{mm';m_1m_1'}(q) = \int \mathrm{d}z_1\,\mathrm{d}z_2\,\psi_m(z_1)\psi_{m'}(z_1)\psi_{m_1}(z_2)\psi_{m_1'}(z_2)\exp(-q|z_1 - z_2|). \tag{8}$$

In (5) we use the notation

$$k^\pm = k \pm q_y/2$$

and

$$\check{F}(q) = \int \frac{\mathrm{d}^2p}{2\pi}F(p)\exp(\mathrm{i}q\cdot p). \tag{9}$$

The appearance in the Hamiltonian (5) of the $\check{V}$-terms is connected with the possibility of representing the exchange interaction as a direct interaction using a Fourier transformation [4, 6].

The functions $L_{n,n'}$ are the same as in the two-dimensional problem [4, 6]. They may be represented through the Laguerre polynomials L_n^m:

$$L_{nn'}(q) = L_n^{n-n'}(q^2/2)\exp(-q^2/4).$$

The influence of the electron motion along the z direction is described by the overlap integrals $J_{ij;kl}(q)$ (equation (8)). These integrals are slow functions of the dimensionless parameter Lq. The behaviour of these functions in the limiting cases can be obtained for any concrete wavefunctions ψ_m. Later the following asymptotic values are used:

$$\begin{aligned} J_{ii;jj}(q) &= 1 - \alpha_{ij} qL && \text{when } qL \ll 1 \\ J_{ij;ij}(q) &= \beta_{ij} qL - \beta_{ij}^{(1)} q^2 L^2 && qL \ll 1;\ i \not\equiv j \\ J_{ii;jj}(q) &= J_{ij;ij}(q) = C_{ij}/qL && \text{when } qL \gg 1. \end{aligned} \tag{10}$$

The numerical coefficients α_{ij}, β_{ij}, $\beta_{ij}^{(1)}$ and C_{ij} depend on the concrete shape of the wavefunctions $\psi_m(z)$. Because of the orthonormality of these wavefunctions it can be shown that all numerical coefficients in (10) are positive. They are equal to

$$\begin{aligned} \alpha_{ij} &= \frac{1}{L}\int \mathrm{d}z_1\,\mathrm{d}z_2\,|\psi_i(z_1)|^2|\psi_j(z_2)|^2|z_1 - z_2| \\ \beta_{ij} &= \frac{1}{L}\int \mathrm{d}z \left(\int^z \psi_i(t)\psi_j(t)\,\mathrm{d}t\right)^2 \\ \beta_{ij}^{(1)} &= \left|\frac{1}{L}\int \mathrm{d}z\,\psi_i(z)\psi_j(z)z\right|^2 \\ C_{ij} &= 2L\int \mathrm{d}z\,|\psi_i(z)|^2|\psi_j(z)|^2. \end{aligned} \tag{11}$$

As mentioned above, every magnetoexciton corresponds to the transition of an electron from the occupied level N_0 to the empty level M_0. The operator of the creation of such a magnetoexciton with momentum p is

$$\hat{A}^+_{M_0N_0}(p) = \sum_k \exp(\mathrm{i}p_x k)\,\hat{b}^+_{M_0k+}\hat{a}_{N_0k-}. \tag{12}$$

This operator commutes with the Hamiltonian (5) for the integer filling factors. To obtain the spectrum of the magnetoexciton, one can find this commutator. Direct calculations give the following result:

$$E_{N_0M_0}(p) = E^{(0)}_{N_0M_0} + E^{(1)}_{N_0M_0} + \Delta E_{N_0M_0}(p) \tag{13}$$

where

$$E^{(0)}_{N_0M_0} = (n_0 - n_1)\hbar\omega_c + E_{m_0} - E_{m_1} \tag{14}$$

$$E^{(1)}_{N_0M_0} = \sum_N \left(\int \frac{\mathrm{d}^2q}{2\pi}[V_{NN;N_0N_0}(q) - V_{M_0M_0;NN}(q)] + \lim_{q=0}[V_{M_0N;M_0N}(q) - V_{N_0N;N_0N}(q)]\right) \tag{15}$$

$$\Delta E_{N_0M_0}(p) = V_{M_0N_0;M_0N_0}(p) - \check{V}_{M_0M_0;N_0N_0}(p). \tag{16}$$

The value $E^{(0)}$ (equation (14)) is the energy of the corresponding one-electron transitions, and E_m is the energy of the size-quantized level m. The other terms in (13) are the result of the many-electron interaction. The p-independent energy shift $E^{(1)}$ (equation (15)) is the contribution to the magnetoexciton energy due to the interaction of the excited electron and the hole with all other electrons in the filled levels. The last term in square brackets in (15) is equal to zero when we study the transition between two different Landau levels, but it gives an important contribution to the energy shift when the transitions with a change in quantum number m are taken into account.

The dispersion of magnetoexcitons is described by the term $\Delta E_{M_0N_0}$ (equation (16)) which depends on the matrix elements for the states N_0 and M_0 only.

The general formula (13) contains all the branches of magnetoexcitons. It is possible to calculate the energy spectrum for any non-degenerate excitation using the corresponding wavefunctions. Besides this, some general results may be obtained for all types of magnetoexciton.

Three different types of magnetoexciton can be distinguished which correspond to the various changes in quantum numbers n and m. These are the following:

(1) the ordinary magnetoexciton, which corresponds to the transition of an electron between two Landau levels without any change in the size quantization state (only this type of magnetoexciton exists in ideal two-dimensional systems);

(2) the magnetoexciton which corresponds to the transition of an electron between two different levels of size quantization without any change in the Landau level; this excitation is called the interlevel magnetoexciton;

(3) the mixed magnetoexciton where both quantum numbers n and m are changed in the electron transition.

These three types of magnetoexciton have qualitatively different dispersion laws. This makes it possible to distinguish between various magnetoexcitons experimentally.

It should be noted that, when more than one energy level is occupied by electrons, some one-electron transitions may have the same energies. The magnetoexcitons which correspond to the electron transitions with the same energies will be called degenerate magnetoexcitons. The main reason for this degeneracy is that the energy differences between the nearest Landau levels are the same. In the next section the conditions for the absence of degeneracy are established, and some general relations for all branches of magnetoexcitons are obtained.

3. Non-degenerate magnetoexcitons

When only one energy level is occupied by electrons, all branches of magnetoexcitons are non-degenerate. For a larger filling factor the conditions for the existence of non-degenerate excitations are different for all types of magnetoexciton.

3.1. Ordinary magnetoexcitons

The spectrum for this type of excitation is non-degenerate when all filled levels correspond to the first level of size quantization and to different Landau levels, and the transition takes place between the nearest Landau levels. The finite size of the quantum well results in some quantitative changes in the spectrum in comparison with the two-dimensional case [3, 4], but the qualitative character of the dispersion

law remains the same. The changes in the spectrum are connected with the softening of the Coulomb interaction for $L \not\equiv 0$ and are quite similar to those obtained for the interband exciton in a quantum well [7]. In a formal way, the softening of the Coulomb interaction is described by the difference between unity and the overlap integral J_{1111} (equation (8)). It should be noted that the energy of the ordinary magnetoexciton at $p = 0$ is equal to the one-electron energy $\hbar\omega_c$ independent of the value of J_{1111}. This is a consequence of the Kohn [8] theorem which states that the energy of cyclotron resonance does not depend on the electron–electron interaction.

For this type of magnetoexciton the general formula for the spectrum according to (13) has the following form:

$$E_{n_0}(p) = \hbar\omega_c + E_0\left(\frac{J_{1111}(p)}{p}|L_{n_0n_0+1}(p)|^2 - \int \frac{d^2q}{2\pi}[\exp(iq\cdot p) - 1]\frac{J_{1111}(q)}{q}L_{n_0n_0}(q)L_{n_0+1,n_0+1}(q)\right). \tag{17}$$

The momentum-dependent part of the energy ΔE_{n_0} in the limiting cases of small and large momenta p are given by

$$\begin{aligned} \Delta E_{n_0}(p) &= E_0(pr_0/\hbar)[(n_0+1)/2] \qquad \text{when } pr_0/\hbar \ll 1 \\ \Delta E_{n_0}(p) &= E_0(B_{n_0} - \hbar/pr_0) \qquad \text{when } pr_0/\hbar \gg 1. \end{aligned} \tag{18}$$

In equations (18) the parameter B_{n_0} depends on the relation between the quantum well width L and the magnetic length r_0. This parameter determines the full width of the spectrum of ordinary magnetoexcitons. The analytical expressions for B_{n_0} can be obtained using the asymptotic behaviour of the overlap integrals J_{1111} (equation (10)). They are

$$\begin{aligned} B_{n_0} &= (\pi/2)^{1/2}\gamma_{n_0} - \alpha_{11}L/r_0 \qquad L \ll r_0 \\ B_{n_0} &= (C_{11}r_0/L)\ln[(L/r_0)t_0] \qquad L \gg r_0 \end{aligned} \tag{19}$$

where the numerical coefficient γ_n is determined as an integral of the Laguerre polynomials L_n:

$$\gamma_n = \left(\frac{2}{\pi}\right)^{1/2}\int dq \exp\left(-\frac{q^2}{2}\right) L_n\left(\frac{q}{2}\right)^2 L_{n+1}\left(\frac{q}{2}\right)^2. \tag{20}$$

For the lowest-energy levels these coefficients are given by: $\gamma_0 = \frac{1}{2}$, $\gamma_1 = \frac{7}{16}$, $\gamma_2 = \frac{51}{128}$,.... The numerical coefficient t_0 which appears in (19) in the logarithm is of the order of unity and may be calculated in a similar way as in the problem of interband excitons [7]. For instance, in the case of a transition between the ground state (0, 1) and the excited state (1, 1) it has the form

$$\ln t_0 = \frac{1}{2}\ln\left(\frac{2}{\gamma}\right) - \frac{1}{2} + \frac{L}{C_{11}}\int dx\,dy\,\psi_1^2(x)\left(\frac{d}{dy}\psi_1^2(y)\right)\operatorname{sgn}(x-y)\ln\left(\frac{|x-y|}{L}\right) \tag{21}$$

where ψ_1 is the wavefunction of the electron at the first level of size quantization. For the rectangular well with infinite barriers this number is $\ln t_0 = -1.6955$.

The dispersion curve $\Delta E(p)$ in the region of intermediate values of p has the qualitative behaviour obtained in [3, 4] for the case $L = 0$. The calculations show

that there is at least one local minimum of the energy at $pr_0/\hbar \simeq 1$. This minimum is a result of the different p-dependences for the direct and exchange Coulomb interaction. The position and relative depth of this minimum have a very slight dependence on the relation between L and r_0. For higher Landau levels, some additional local minima may occur owing to the oscillations of the functions $L_{nm}(q)$ (equation (7)), but as in the two-dimensional case [4] these additional minima are very narrow and it is very difficult to observe them experimentally. Later, only the main energy minimum is taken into consideration. In fact, for this type of magnetoexciton the finite size of the quantum well results only in a decrease in the scale of the electron–electron interaction energy, which is described by the parameter B_{n_0} (equation (19)).

3.2. *Interlevel magnetoexcitons*

In a quantum well with the exception of the specially produced parabolic wells the energy differences between the size quantization levels are not the same. Therefore in principle, non-degenerate interlevel magnetoexcitons which correspond to any change in quantum number m may exist. When all filled levels belong to the first Landau level, all interlevel magnetoexcitons are non-degenerate. However, when the electrons occupy some different Landau levels, then some of the interlevel magnetoexcitons may be degenerate.

In accordance with the general formula (13) the energy of magnetoexcitons corresponding to the transition from the filled level (n_0, m_1) to the empty level (n_0, m_2) may be represented as follows:

$$E_{m_1m_2n_0} = (E_{m_2} - E_{m_1}) + E^{(1)}_{m_1m_2n_0} + \Delta E_{m_1m_2n_0}(p) \tag{22}$$

where

$$\Delta E_{m_1m_2n_0}(p) = E_0\left(\frac{\hbar}{pr_0}|L_{n_0n_0}(p)|^2 J_{m_1m_2;m_1m_2}(p) - \int \frac{d^2q}{2\pi}[\exp(iq\cdot p) - 1]|L_{n_0,n_0}(p)|^2 \frac{J_{m_1m_1;m_2m_2}(q)}{q}\right). \tag{23}$$

It can be seen from (22) and (23) that the many-electron correction $E^{(1)}_{m_1m_2n_0} + \Delta E_{m_1m_2n_0}(0)$ to the one-electron energy is not equal to zero in contrast with the ordinary magnetoexciton (equation (17)). This correction has the same order of magnitude as E_0L/r_0 and it is much smaller than the differences between the one-electron energies due to the parameters (1) and (2). However, this correction as is shown below may be larger than the full width of the spectrum in the case of quite thick quantum wells.

The dispersion law has the following behaviour in the limiting cases:

$$\begin{aligned} &\Delta E_{m_1m_2n_0}(p) = (E_0L/r_0)(\beta_{m_1m_2} - \beta^{(1)}_{m_1m_2}Lp/\hbar) && p < \min\{\hbar/L; \hbar/r_0\} \\ &\Delta E_{m_1m_2n_0} = E_0(B_{m_1m_2n_0} - \hbar/pr_0) && p > \max\{\hbar/L; \hbar/r_0\}. \end{aligned} \tag{24}$$

The numerical coefficients β_{ij} and $\beta^{(1)}_{ij}$ are determined in (11). The dimensionless parameter $B_{m_1m_2n_0}$ depends on the relation between L and r_0, in a similar way to the parameter B_{n_0} (equation (19)). For different values of the ratio L/r_0 this dependence is equal to

$$\begin{aligned} &B_{m_1m_2n_0} = [(\pi/2)^{1/2}\check{\gamma}_{n_0} - \alpha_{m_1m_2}L/r_0] && L \ll r_0 \\ &B_{m_1m_2n_0} = (C_{m_1m_2}r_0/L)\ln[(L/r_0)t_{n_0m_1m_2}] && L \gg r_0 \end{aligned} \tag{25}$$

where

$$\check{\gamma}_{n_0} = \left(\frac{2}{\pi}\right)^{1/2} \int \mathrm{d}q \exp\left(-\frac{q^2}{2}\right) \left[L_{n_0}\left(\frac{q^2}{2}\right)\right]^2 \tag{26}$$

and for the first few Landau levels the numerical coefficients $\check{\gamma}_n$ are $\check{\gamma}_0 = 1$, $\check{\gamma}_1 = \frac{3}{4}$, $\check{\gamma}_2 = \frac{41}{64}$, ... The numerical coefficient $t_{m_1 m_2 n_0}$ in (25) may be expressed in a similar way to the coefficient t_{n_0} (equation (21)) using the wavefunctions ψ_m instead of ψ_1.

It can be seen from (24) that in the region of small momenta p the energy of an interlevel magnetoexciton decreases with increasing p. In the region of large momenta the energy increases with increasing p. Therefore at some finite value of momentum, $p = p_0$, the energy has an absolute minimum. The position and the depth of this minimum depend on the relation between L and r_0. The value of p_0 is equal to $\hbar L^2/r_0^3$ when $L < r_0$ and $\hbar/L$ when $L > r_0$. In the last case the energy differences are given by $E(0) - E(p_0) \simeq E_0$ and have an order of magnitude comparable with the full width of the spectrum.

For $p > p_0$ the energy of an interlevel magnetoexciton increases slowly with increasing momentum. This kind of dispersion is quite similar to that obtained for the σ^+ spin exciton in [4].

The existence of the absolute minimum of the energy at a finite value of p must produce an unusual interaction between the interlevel magnetoexciton and phonons and impurities. We may expect this kind of excitation to have a larger linewidth in the optical spectrum than other branches of magnetoexcitons do.

3.3. Mixed magnetoexcitons

All excitations of this type are non-degenerate when the filled levels belong to the first Landau level. When more than one Landau level is occupied by the electrons, then non-degenerate excitations may also exist. They occur for transitions of the electron from the filled level with a maximum quantum number n to the first Landau level with an arbitrary change in the quantum number m.

As in the case of interlevel magnetoexcitons the electron–electron interaction gives non-zero corrections to the one-particle energy levels. These corrections are of the order of $E_0 L/r_0$. The main characteristic feature of the spectrum of mixed magnetoexcitons is the stronger p-dependence in the region of small momentum. For the magnetoexciton which corresponds to the transition of an electron from the state (n_0, m_0) to the state $(n_1 m_1)$ this dependence according to (16) is

$$\Delta E_{n_0 m_0 n_1 m_1}(p) = E_0 [\![(\beta_{m_0 m_1} L/r_0)(p r_0/\hbar)^{2\Delta n} \{ n!/2^{\Delta n} n_1! [(n-n_1)!]^2 \} - s(p r_0/\hbar)^2]\!] \qquad p < \min\{\hbar/L, \hbar/r_0\} \tag{27a}$$

where $\Delta n = n_0 - n_1$ is the change in the Landau level quantum number. The numerical factor in (27*a*) is written for the case $n_0 > n_1$. The dimensionless parameter s is of the order of unity when $L < r_0$ and of the order of $(r_0/L)^2 \ln(L/r_0)$ when $L > r_0$.

For sufficiently large values of momentum p the usual dependence of the dispersion law on p is realized:

$$\Delta E_{n_0 m_0 n_1 m_1}(p) = E_0 (B_{n_0 m_0 n_1 m_1} - \hbar/p r_0) \tag{27b}$$

where the parameter $B_{n_0 m_0 n_1 m_1}$ depends on the ratio L/r_0 in a similar way to (19) and (25).

In the region of intermediate momenta $p r_0/\hbar \simeq 1$, the dispersion curve has a local minimum similar to that in the spectrum of ordinary magnetoexcitons.

4. Degenerate magnetoexcitons

As mentioned above, the main reason for the existence of degenerate magnetoexcitons is that the differences between the energies of nearest Landau levels are the same. It should be noted that this causes the degeneration of magnetoexcitons of the same type.

To find the spectrum of the degenerate magnetoexcitons, one can calculate the commutator of the Hamiltonian (5) with the linear combination of the excitonic operators (12). As a result a system of linear equations can be obtained. This system contains main terms similar to (13) and some additional terms which describe the interaction of different magnetoexcitons. In the case of twofold degeneracy of magnetoexcitons with quantum numbers M_0, N_0 and M_1, N_1 these additional terms are

$$\delta E_{N_0M_0N_1M_1} = V_{M_0N_0;N_1M_1}(p) - \int \frac{d^2q}{2\pi} \exp(iq \cdot p) V_{M_0M_1;N_0N_1}(q). \tag{28}$$

The interaction (28) removes the degeneracy according to the common rules of quantum mechanics. The calculation shows that for magnetoexcitons of the same type the interaction (28) does not produce any qualitative changes in the spectrum. The most important role is played by the interaction energy (28) at $p = 0$ which gives the splitting $\delta E_{M_0N_0;M_1N_1}(0)$ of the energy gap of magnetoexcitons. This splitting has the order of magnitude

$$\begin{aligned} \delta E &\simeq E_0(L/r_0) \qquad \text{when } L \ll r_0 \\ \delta E &\simeq E_0(r_0/L) \qquad \text{when } L \gg r_0. \end{aligned} \tag{29}$$

The splitting (29) is smaller than the full width of the spectrum (19) or (25) although in a thick quantum well this is a logarithmic function. This is the consequence of the orthogonality of the wavefunctions from the different one-electron levels. As a result the overlap integrals in (28) are smaller than those in (17) and (23).

The position and the relative depth of the energy minimum in dispersion curves depend on the interaction (28) very slightly and they can be described by the equation for non-degenerate magnetoexcitons (equation (16)) with good accuracy.

Apart from the degeneracy of the energies of the same type of magnetoexciton considered above, it is possible that there is an occasional energy degeneracy of different types of magnetoexciton. This occurs when the energy difference between two levels of size quantization is proportional to the cyclotron energy. The spectrum in this case depends essentially on the exact relations between the one-electron energy levels. To obtain the spectrum of interacting magnetoexcitons it is necessary to solve the system of linear equations with interaction (28). This has to be done for every special quantum well structure. In a general case it may be noted that the interacting energy cannot exceed the value for one type of magnetoexciton (equation (29)).

5. Discussion

For the new types of magnetoexciton, namely interlevel and mixed magnetoexcitons, considered above, both the energy gap and the dispersion law depend on the many-particle interactions. Both these values have to vary with the magnetic field and

concentration of the electrons. This makes it possible to use optical measurements in order to obtain additional information on the many-particle correlations in the system under consideration.

A similar approach can be applied to the investigation of the spin exciton spectrum. In a finite-size quantum well there are a number of different types of spin exciton corresponding to various combinations of quantum numbers n, m and s. The main difference in the calculation of the spectrum of spin excitons is the following. The first term in equation (16) caused by the exchange interaction is equal to zero for all types of spin exciton. As a result the dispersion curves have to be different from those for magnetoexcitons, especially in the region of small momenta. Nevertheless the full width of the spectrum of spin excitons must decrease with increase in the quantum well width because of the softening of Coulomb interaction as in the case of magnetoexcitons. This may be important, for instance, for the better understanding of recent experimental results on nuclear-spin relaxation [9]. The theoretical approach based on the properties of two-dimensional spin excitons was successfully used to describe these experiments [10].

Another possible application of the results obtained is the theory of magnetoluminescence of the quantum wells with non-equilibrium electrons and holes created by optical pumping. In the paper by Bychkov and Rashba [11] the concept of two-dimensional magnetoexcitons was used to describe the available experimental data [12, 13]. Qualitative agreement between theory and experiment was obtained.

However, the experiments [12, 13] show different dependences of the luminescence lines on the concentration of carriers in various quantum well structures. These data have no obvious explanation at present. The discrepancy between experimental results in [12] and in [13] may be understood by taking into consideration the additional branches of magnetoexcitons. This work is now in progress.

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